Addendum to paper on 'A two-dimensional analogue of Padé approximants'

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ADDENDUM

Addendum to paper on 'A two-dimensional analogue of Padé approximants'

C H Lutterodt
Department of Mathematics, University of Cape Coast, Cape Coast, Ghana

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Abstract. We state and prove a theorem about the fact that B^1-type approximants have reciprocals with different boundary structure from their own.

In this brief follow-up note to the paper by Lutterodt (1974) we report on a property of the B^1-type approximants discussed in that paper. We state this property in a form of a theorem and prove the result.

Theorem

Reciprocals of the B^1-type approximants to a Taylor expansion of a holomorphic function do not have a B^1-type boundary.

Proof

Write

$$\frac{1}{f(\zeta)}-\frac{1}{R_{NN';MM'}(\zeta)}=\frac{R_{NN';MM'}(\zeta)-f(\zeta)}{f(\zeta)R_{NN';MM'}(\zeta)}.$$

When $f(\zeta)\neq 0$ (because $C_{00}\neq 0$), $R_{NN';MM'}(\zeta)\neq 0$ in some neighbourhood of $(0,0)$. Definition of B^1-type approximant implies

$$\frac{1}{R_{NN';MM'}(\zeta)}-\frac{1}{f(\zeta)}=\frac{\tilde{B}^1_{NN';MM'}(\zeta)}{P_{NN'}(\zeta)}+O(z_1^{N_1+M_1+1})O(z_2^{N_2+M_2+1})$$

where $\tilde{B}^1_{NN';MM'}(\zeta)=B^1_{NN';MM'}(\zeta)/f(\zeta)$. By writing

$$\frac{1}{f(\zeta)}=\sum_{\alpha,\beta=0}^{\infty}d_{\alpha\beta}z_1^{\alpha}z_2^{\beta}$$

we see that up to $O(z_1^{N_1+M_1+1})O(z_2^{N_2+M_2+1})$, $\tilde{B}_{NN';MM'}(\zeta)$ takes the form

$$\tilde{B}^1_{NN';MM'}(\zeta)=\sum_{\alpha=1}^{N+M}\sum_{\beta=N'+1}^{N'+M'}z_1^{\alpha}z_2^{\beta}\left(\sum_{\lambda_1=1}^{\min(N'\alpha)}\sum_{\lambda_2=N'+1}^{\beta}A_{\lambda_1\lambda_2}d_{\alpha-\lambda_1,\beta-\lambda_2}\right)$$
$$+\sum_{\alpha=N+1}^{N+M}\sum_{\beta=1}^{N'+M'}z_1^{\alpha}z_2^{\beta}\left(\sum_{\lambda_1=N+1}^{\alpha}\sum_{\lambda_2=1}^{\min(N',\beta)}A_{\lambda_1\lambda_2}d_{\alpha-\lambda_1,\beta-\lambda_2}\right)$$

where

$$A_{\lambda_1\lambda_2} = \sum_{r_1=0}^{\min(M,\lambda_1)} \sum_{r_2=0}^{\min(M',\lambda_2)} b_{r_1r_2}c_{\lambda_1-r_1,\lambda_2-r_2}$$

and

$$B^1_{NN';MM'}(\zeta) = \sum_{\alpha=1}^{N} \sum_{\beta=N'+1}^{N'+M'} z_1^\alpha z_2^\beta A_{\alpha\beta} + \sum_{\alpha=N+1}^{N+M} \sum_{\beta=1}^{N'} z_1^\alpha z_2^\beta A_{\alpha\beta}.$$

Now $\tilde{B}^1_{NN';MM'}(\zeta)$ is structurally different from $B^1_{NN';MM'}(\zeta)$ and since it is the structure of these B's which determines the boundary of the approximant, the approximant of f must have a different boundary from that of f^{-1} if the latter exists.

That the boundary of the reciprocal B^1 approximant is different from that of the B^1 approximant itself is not a serious restriction. It does not, however, tally with the Padé approximant case where the approximant and its reciprocal share an identical boundary, which is a single point. More generally rational approximants in several variables which share the same boundary with their reciprocal, in agreement with the one-dimensional Padé case, have been constructed (see Chisholm 1973, John and Lutterodt 1973, Lutterodt 1974, Chisholm and McEwan 1974, Graves-Morris *et al* 1974, Hughes-Jones 1973).

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